

# **Proposed Moduli of Dry Rock and Their Application to Predicting Elastic Velocities of Sandstones**

Scientific Investigations Report 2005–5119

**U.S. Department of the Interior**  
**U.S. Geological Survey**

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By Myung W. Lee

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# Proposed Moduli of Dry Rock and Their Application to Predicting Elastic Velocities of Sandstones

By Myung W. Lee

## Abstract

Velocities of water-saturated isotropic sandstones under low frequency can be modeled using the Biot-Gassmann theory if the moduli of dry rocks are known. On the basis of effective medium theory by Kuster and Toksöz, bulk and shear moduli of dry sandstone are proposed. These moduli are related to each other through a consolidation parameter and provide a new way to calculate elastic velocities. Because this parameter depends on differential pressure and the degree of consolidation, the proposed moduli can be used to calculate elastic velocities of sedimentary rocks under different in-place conditions by varying the consolidation parameter. This theory predicts that the ratio of P-wave to S-wave velocity ( $V_p/V_s$ ) of a dry rock decreases as differential pressure increases and porosity decreases. This pattern of behavior is similar to that of water-saturated sedimentary rocks. If microcracks are present in sandstones, the velocity ratio usually increases as differential pressure increases. This implies that this theory is optimal for sandstones having intergranular porosities. Even though the accurate behavior of the consolidation parameter with respect to differential pressure or the degree of consolidation is not known, this theory presents a new way to predict S-wave velocity from P-wave velocity and porosity and to calculate elastic velocities of gas-hydrate-bearing sediments. For given properties of sandstones such as bulk and shear moduli of matrix, only the consolidation parameter affects velocities, and this parameter can be estimated directly from the measurements; thus, the prediction of S-wave velocity is accurate, reflecting in-place conditions.

## Introduction

In order to relate elastic velocities (P-wave velocity,  $V_p$ , and shear-wave velocity,  $V_s$ ) to physical properties of sedimentary rocks, an accurate forward modeling is required. Elastic velocities are generally dispersive and depend on many factors such as porosity, differential pressure, and degree of consolidation. Biot (1956) developed theoretical formulas for predicting the frequency-dependent elastic velocities for water-saturated rocks for all ranges of frequency, and Geertsma and Smit

(1961) analyzed the Biot equation for dilatational waves in the low-frequency range. Gassmann (1951) derived equations for elastic velocities of the fluid-saturated porous media at zero-frequency. The Gassmann (1951) theory predicts the bulk modulus of the fluid-saturated porous medium from the known bulk moduli of the solid matrix, the frame, and the pore fluid and assumes that the shear modulus of sedimentary rocks is not affected by fluid saturation. The essential input for the application of the Gassmann equation is the bulk and shear moduli of the dry frame, which are usually measured in the laboratory or calculated using theories such as a contact theory (Digby, 1981; Murphy and others, 1993; Winkler, 1983).

The velocity ratio,  $V_p/V_s$ , is a useful parameter to evaluate the characteristics of velocity models of dry rocks. Pickett (1963) demonstrated that  $V_p/V_s$  of the partially gas-saturated sediment are almost constant, irrespective of porosity. Based on this observation, Krief and others (1990) derived the shear modulus of dry rock, which is a simple function of the Biot coefficient similar to the bulk modulus predicted from the Biot theory. Krief's theory predicts a constant  $V_p/V_s$  for a dry rock irrespective of porosity and differential pressure. The contact theory predicts that  $V_p/V_s$  decreases as differential pressure increases and is independent of porosity. The Kuster and Toksöz (1974) theory (KTT), which was developed based on wave scattering theory, provides moduli of the dry frame and predicts that  $V_p/V_s$  increases as porosity increases.

Recently, Pride (2005) presented bulk and shear moduli (which are functions of porosity and a consolidation parameter) of dry sandstones. The consolidation parameter depends on differential pressure and the degree of consolidation. This theory predicts that the velocity ratio,  $V_p/V_s$ , decreases as differential pressure increases and porosity decreases. MacBeth (2004) derived an empirical relation of moduli with respect to differential pressure based on the theory by Sayers and Kachanov (1995) and demonstrated that, in general,  $V_p/V_s$  decreases as differential pressure increases, mainly because of microcracks present in sandstones.

An advantage of using moduli derived by Pride (2005) is that there is a consolidation parameter both in bulk and shear moduli, and this parameter can be treated as a free parameter in predicting velocities or matching observed velocities. Therefore, moduli recommended by Pride (2005) provide an accurate method of predicting S-wave velocities from P-wave

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velocity and porosity. Many approaches exist for predicting S-wave velocity. For example, (1) Greenberg and Castagna (1992) proposed a semiphysical model based on the Biot-Gassmann theory (BGT); (2) Xu and White (1996) predicted S-wave velocity by a combination of Kuster and Toksöz (1974) theory and differential effective medium theory (Cheng and Toksöz, 1979), using pore aspect ratios to characterize compliance of sand and clay components; (3) Lee (2003) predicted S-wave velocity by using the modified BGT under the assumption that  $V_p/V_s$  is a function of porosity. Also, many empirical methods are used (for example, Castagna and others, 1985; Han and others, 1986; Wang, 2000). Because there are no other input parameters except P-wave velocity and porosity when using the moduli by Pride, the prediction of S-wave velocity is unbiased and simple.

Elastic velocities of gas-hydrate-bearing sediments have been investigated by many researchers (for example, Carcione and Tinivella, 2000; Helgerud, 2001; Jakobsen and others, 2002; Lee, 2002). Assuming a pore-filling model of gas hydrate in the pore, the proposed equation by Pride (2005) provides a new way of calculating elastic velocities of gas-hydrate-bearing sediments.

In this paper, moduli of dry rocks similar to those recommended by Pride (2005) are derived to predict elastic velocities of water-saturated sedimentary rocks having different in-place conditions under the assumption that velocity dispersion, attenuation, and anisotropy can be ignored. This theory is applied to consolidated sediments measured by Han and others (1986), semiconsolidated sediments by Gregory (1976), and unconsolidated sediments by Hamilton (1971) and Domenico (1977). This theory is also applied to well-log velocities for unconsolidated sediments acquired at the Alpine-1 well on the North Slope of Alaska and at the Mallik 5L-38 well, Mackenzie Delta, in Canada. The applied theory results in good agreement between measured and predicted velocities.

## Theory

### Velocity Equation

Elastic velocities (that is, compressional [P]-wave velocity [ $V_p$ ] and shear [S]-wave velocity [ $V_s$ ]) of water-saturated sedimentary rocks can be computed from the elastic moduli by the following formulas:

$$V_p = \sqrt{\frac{k + 4\mu/3}{\rho}} \text{ and } V_s = \sqrt{\frac{\mu}{\rho}} \quad (1)$$

where  $k$ ,  $\mu$ , and  $\rho$  are bulk modulus, shear modulus, and density of the water-saturated sedimentary rocks, respectively. The formation density is given by

$$\rho = (1 - \phi)\rho_{ma} + \phi\rho_{fl}, \quad (2)$$

where  $\phi$ ,  $\rho_{ma}$ , and  $\rho_{fl}$  are porosity, matrix density, and pore-fluid density, respectively. The bulk and shear moduli of a composite matrix—for example, quartz and clay—are calculated using Hill's averaging method (1952).

Under the low-frequency approximation, the classical Biot-Gassmann theory (BGT) (Biot, 1941, 1956; Gassmann, 1951) predicts bulk modulus of water-saturated rocks from the following equations, if the Biot coefficient ( $\beta$ ) of the dry frame is known.

$$k = k_{ma}(1 - \beta) + \beta^2 M, \quad (3)$$

where

$$\frac{1}{M} = \frac{(\beta - \phi)}{k_{ma}} + \frac{\phi}{k_f}$$

and  $k$  is the bulk modulus of water-saturated sediments, and  $k_{ma}$  and  $k_f$  are bulk modulus of matrix (constitutes the skeleton of the formation) and bulk modulus of fluid, respectively.

The Biot coefficient is defined as  $\beta = 1 - k_d/k_{ma}$ , where  $k_d$  is the dry-rock bulk modulus. The Biot coefficient is only defined for the bulk modulus because it is the ratio of pore-volume change to total bulk-volume change under dry or drained conditions. However, the BGT does not provide a relation between the shear modulus and matrix material using the Biot coefficient.

### Moduli of Dry Frame

Within the poroelastic framework, dry moduli of the frame are undetermined and must be specified a priori. Generally, moduli are measured in the laboratory (Murphy, 1984), are predicted by theory—for example, contact theory by Digby (1981)—or can be derived under a specific assumption. If it is assumed that the velocity ratio,  $V_p/V_s$ , is constant irrespective of porosity, which is approximately true for partially gas saturated sedimentary rocks (Pickett, 1963), and is equal to the velocity ratio of matrix, the shear modulus of dry rock ( $\beta$ ) can be derived by the following formula (Krief and others, 1990):

$$\mu_d = \mu_{ma}(1 - \beta) \quad (4)$$

Conventionally, it is assumed that fluid in the pore space does not change the shear modulus of sedimentary rocks. In other words, the shear modulus of dry rock is the same as that of water-saturated sedimentary rock or  $\mu = \mu_d$ , where  $\mu$  is the shear modulus of the water-saturated rock. Therefore, under this assumption, equation 4 can also be used for water-saturated rock. As shown in equations 1–4, by using the Gassmann theory, velocities of water-saturated sedimentary rocks can be obtained if moduli of the dry frame are known.

Pride (2005) provided the following dry-frame moduli for consolidated sandstones:

$$k_d = \frac{k_{ma}(1-\phi)}{(1+\alpha\phi)} \quad (5)$$

$$\mu_d = \mu = \frac{\mu_{ma}(1-\phi)}{(1+1.5\alpha\phi)} \quad (6)$$

where  $\alpha$  is a consolidation parameter that represents the degree of consolidation between grains. Effective medium theories can be approximately manipulated into expressions of this form and predict that  $\alpha$  depends on both the shape of the cavity and the ratio  $\mu_{ma}/k_{ma}$  (Pride, 2005). The factor 1.5 in equation 6 (2 or 5/3 are also reasonable) is somewhat arbitrary but yields an accurate  $V_p/V_s$  ratio for sandstones.

In order to generalize the Pride equation, the shear modulus of dry rock is proposed in the following equation:

$$\mu_d = \mu = \frac{\mu_{ma}(1-\phi)}{(1+\gamma\alpha\phi)} \quad (7)$$

where

$$\gamma = \frac{1+2\alpha}{1+\alpha}. \quad (8)$$

Note that when  $\alpha = 1$ ,  $\gamma = 1.5$ , which is identical to equation 6. When  $\alpha = 2$ , then  $\gamma = 5/3$ , and as  $\alpha$  increases,  $\gamma$  approaches 2. Therefore, equation 8 covers all reasonable values suggested by Pride (2005).

In order to relate equations 5 and 7 to equations 3 and 4, the following coefficients are defined using equations 5 and 7:

$$\beta_p \equiv 1 - \frac{k_d}{k_{ma}} = \frac{\phi(1+\alpha)}{(1+\alpha\phi)} \Rightarrow k_d = k_{ma}(1-\beta_p) \quad (9)$$

$$\beta_s \equiv 1 - \frac{\mu_d}{\mu_{ma}} = \frac{\phi(1+\gamma\alpha)}{(1+\gamma\alpha\phi)} \Rightarrow \mu = \mu_{ma}(1-\beta_s). \quad (10)$$

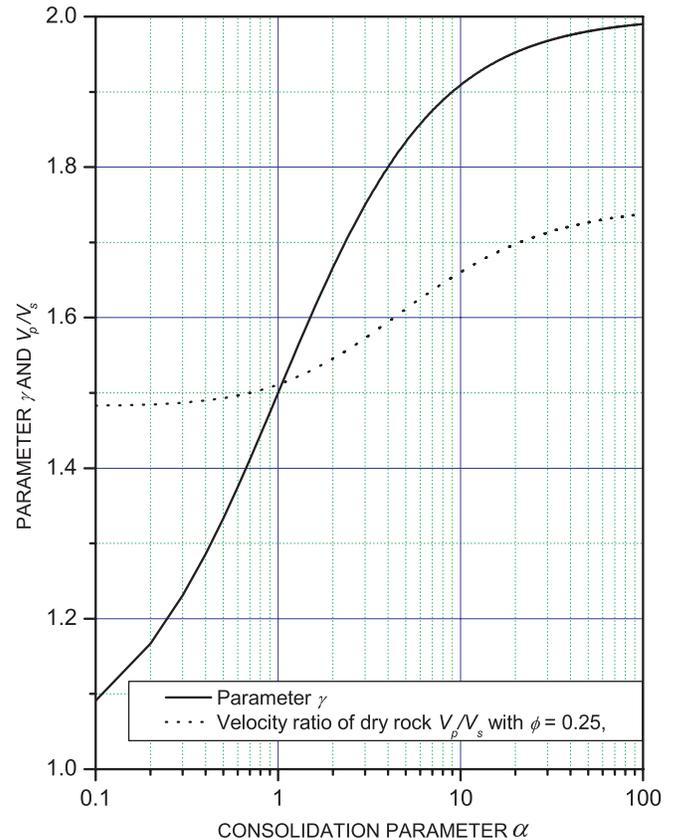
Parameter  $\beta_p$ , equation 9, has the real meaning of the Biot coefficient, but  $\beta_s$ , equation 10, is just a convenient parameter to put the shear modulus in the same form as the bulk modulus. It is noted that  $\beta_p$  is always less than or equal (when  $\alpha = 0$ ) to  $\beta_s$  for a given degree of consolidation and porosity and is the same as  $\beta_s$  at  $\phi = 0$  and  $\phi = 1$  for all values of  $\alpha$ .

In the framework of Pride (2005), velocities of sandstones for a given porosity are determined only by the consolidation parameter  $\alpha$ . For sandstones,  $\alpha$  lies in the approximate range  $2 < \alpha < 20$  (Pride, 2005). As the Biot coefficient decreases, moduli of the dry frame increase. Therefore, as  $\alpha$  decreases, moduli (or velocities) increase. In this new formulation, velocities of sandstones for a given porosity are determined only by the parameter  $\alpha$ , which could vary between 0 and infinity.

Figure 1 shows relation between parameters  $\alpha$  and  $\gamma$ . At  $\alpha = 0$ ,  $\gamma = 1$ , and at  $\alpha = 100$ ,  $\gamma \approx 1.99$ . As parameter  $\alpha$  increases from 0 to 10, parameter  $\gamma$  increases rapidly and approaches asymptotically to  $\gamma = 2$  as  $\alpha$  further increases.

The dotted line in figure 1 shows the velocity ratio of dry rock ( $V_p/V_s$ ) having  $\phi = 0.25$  with elastic constants shown in table 1. For all ranges of  $\alpha$ , the velocity ratio varies between 1.48 and 1.74. The velocity ratio is insensitive to variation in porosity. The velocity ratios determined by Domenico (1977) for unconsolidated sands with  $\phi = 0.382$ , and by Gregory (1976) for semiconsolidated sandstones with  $\phi = 0.217$ , fall within this range for differential pressure greater than 10 megapascals (MPa).

Figure 2 shows the calculated Biot coefficients with respect to parameter  $\alpha$  for a clean sandstone having porosity of 0.25 using the elastic constants shown in table 1. As indicated in figure 2, as  $\alpha$  increases, both  $\beta_p$  and  $\beta_s$  increase, which means that velocities decrease. Also note that  $\beta_s$  is always greater than  $\beta_p$ . The behavior of the Biot coefficients indicates that the dependence of velocity on differential pressure can be modeled by decreasing parameter  $\alpha$  as differential pressure increases. Figure 2 also demonstrates that velocities



**Figure 1.** Relation between consolidation parameter ( $\alpha$ ) and velocity ratio (solid line) and relation between consolidation parameter ( $\alpha$ ) and parameter  $\gamma$ .  $\phi$ , porosity;  $V_p$ , P-wave velocity;  $V_s$ , S-wave velocity.

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**Table 1.** Elastic constants used for this study.

[GPa, gigapascals; kg/m<sup>3</sup>, kilograms per cubic meter]

Parameters	Values used	Sources
Shear modulus of quartz	44 GPa	Carmichael (1989)
Bulk modulus of quartz	38 GPa	Carmichael (1989)
Shear modulus of clay	6.85 GPa	Helgerud and others (1999)
Bulk modulus of clay	20.9 GPa	Helgerud and others (1999)
Bulk modulus of water	2.29 GPa	
Density of quartz	2,650 kg/m <sup>3</sup>	Helgerud and others (1999)
Density of clay	2,580 kg/m <sup>3</sup>	Helgerud and others (1999)

for consolidated rocks and unconsolidated sediments can be modeled using different values of  $\alpha$ , where  $\alpha$  for the consolidated rock is less than that of the unconsolidated sediment.

For comparison, the Biot coefficient using the Pride equation (equation 6) is shown as dotted line in figure 2. The Biot coefficient of S-wave using the Pride equation is greater than that from the proposed equation (equation 7) when  $\alpha < 1$  and is less than that from the proposed equation when  $\alpha > 1$ . This implies that equation 7 predicts lower velocities than those predicted from the Pride equation for most rocks for a given porosity.

### Comparison with MacBeth Equation

The moduli defined in equations 5 and 7 are similar to those defined by MacBeth (2004):

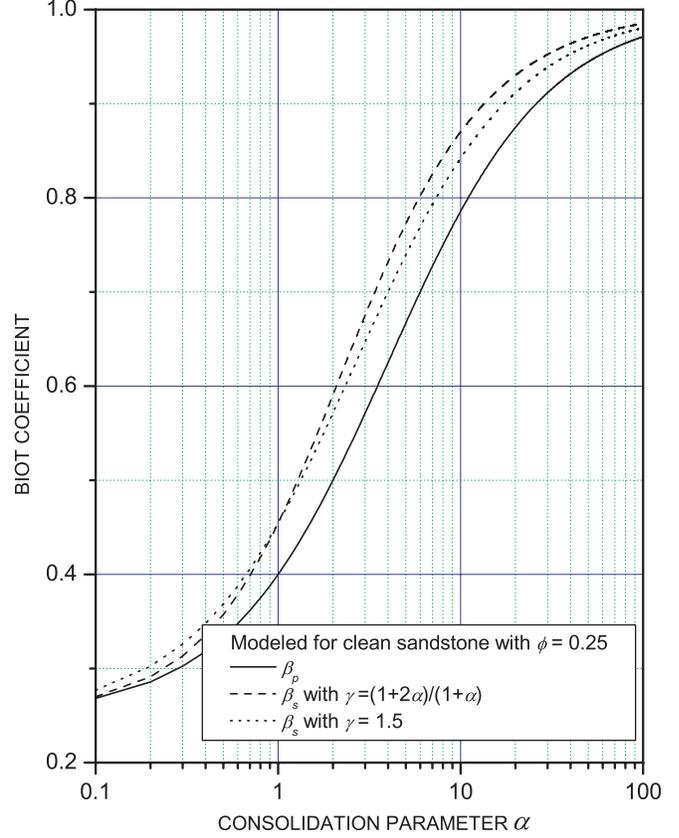
$$k_d(p) = \frac{k_d^\infty}{1 + E_k e^{-p/p_k^*}} \quad (11)$$

where  $p$  is differential pressure and  $k_d^\infty$  is the bulk modulus of dry rock at infinite differential pressure. Let us assume that the consolidation parameter is given by  $\alpha = \alpha_o + \alpha_1 e^{-p/p_k^*}$ . Note that  $\alpha_o$  is the consolidation parameter at infinite pressure and  $(\alpha_o + \alpha_1)$  is the consolidation parameter at  $p = 0$ . Then, the bulk modulus given in equation 5 can be written as

$$k_d(p) = \frac{k_{ma}(1 - \phi)}{(1 + \phi\alpha_o)(1 + E_k e^{-p/p_k^*})} \quad (12)$$

where

$$E_k = \frac{\phi\alpha_1}{1 + \phi\alpha_o} \quad (13)$$



**Figure 2.** Relation between consolidation parameter ( $\alpha$ ) and the Biot coefficient for a rock having porosity ( $\phi$ ) = 0.25 and clay volume content ( $C_v$ ) = 0.  $\beta_p$  and  $\beta_s$  are the Biot coefficients for the P-wave velocity and S-wave velocity, respectively.

Comparing equation 11 with equation 12,

$$k_d^\infty = \frac{k_{ma}(1 - \phi)}{1 + \phi\alpha_o} \quad (14)$$

Equation 14 indicates that the bulk modulus at high differential pressure decreases as porosity and  $\alpha$  increase. Likewise, the shear modulus defined by MacBeth (2004), which is

$$\mu_d(p) = \frac{\mu_d^\infty}{1 + E_\mu e^{-p/p_\mu^*}} \quad (15)$$

can be related to equation 7. The relation is

$$\mu_d^\infty = \frac{\mu_{ma}(1 - \phi)}{1 + \gamma\phi\alpha_o} \quad (16)$$

$$E_\mu = \frac{\gamma\phi\alpha_1}{1 + \gamma\phi\alpha_o} \quad (16)$$

In the framework of this model,  $p_k^* = p_\mu^*$  and  $E_\mu \geq E_k$ . The velocity ratio,  $V_p/V_s$ , is given by

$$\frac{V_p}{V_s} = \sqrt{\frac{4}{3} + \frac{k}{\mu}}.$$

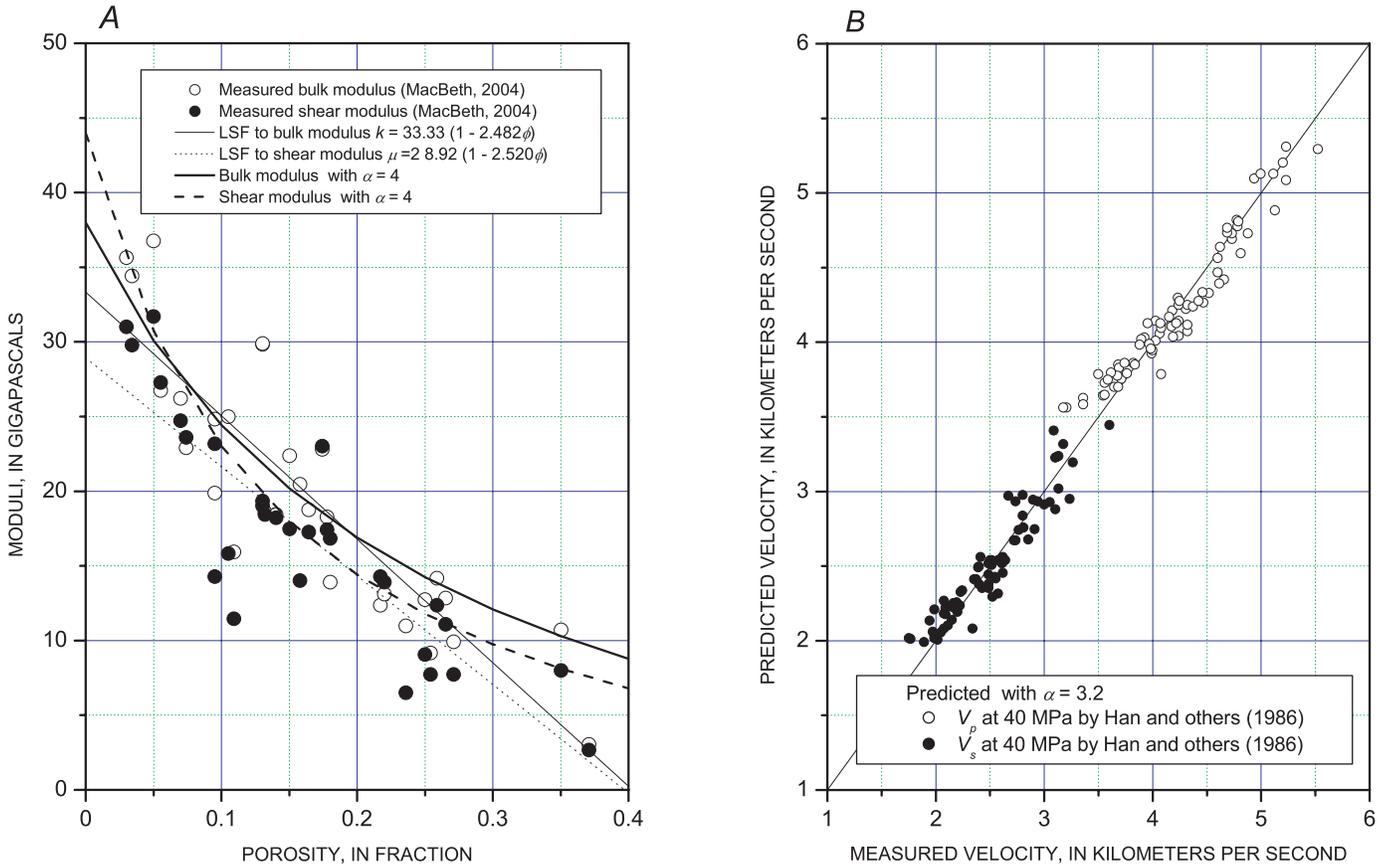
Therefore, the velocity ratio can be deduced from the ratio of moduli ( $k/\mu$ ), which is given by the following equation for a dry rock:

$$\frac{k^d}{\mu^d} = \frac{(1 + \gamma\phi\alpha_o)(1 + E_\mu e^{-p/p^*})}{(1 + \phi\alpha_o)(1 + E_k e^{-p/p^*})} \quad (17)$$

where  $p^* = p_k^* = p_\mu^*$ . Equation 17 implies that  $V_p/V_s$  ratio increases as porosity and the consolidation parameter increase, and the ratio decreases as differential pressure increases. This characteristic is different from the general behavior of sandstones investigated by MacBeth (2004). The  $V_p/V_s$  shown in equation 17 agrees well with that of water-saturated sedimentary rocks (Lee, 2003).

Figure 3A shows the relation between porosity and moduli of dry frame at high differential pressure (that is,  $k_d^\infty$ ), and figure 3B shows the derived Biot coefficients using equations 9 and 10. The large dots and circles in figure 3A are moduli at infinite differential pressure shown in table 2 of MacBeth (2004). Thin solid and dotted lines represent the least-squares fit (LSF) curves of the data. Thick solid and dashed lines are calculated moduli for a clean sandstone using equations 5 and 7 and elastic constant shown in table 1 with  $\alpha = 4$ . The difference between the moduli at zero porosity derived from this study and that of MacBeth is partly caused by clay content in the measured samples. Figure 3A indicates that the moduli predicted by the proposed equation deviated more from that by MacBeth as porosity increases beyond about 0.3.

Figure 3B shows the calculated Biot coefficients using equations 9 and 10 for the result shown in figure 3A. The Biot coefficients for the MacBeth data are calculated from the LSF curves, and the bulk and shear moduli of matrix used for MacBeth data are those at zero porosity shown in figure 3B. The Biot coefficient derived from dry bulk moduli ( $\beta_p$ ) is always less than that derived from the shear moduli ( $\beta_s$ ), even though



**Figure 3.** Relations between dry-rock properties with respect to porosity. *A*, Relation between dry-rock moduli and porosity. LSF, least-squares fit. *B*, Relation between the Biot coefficient and porosity. The Biot coefficient derived by Krief and others (1990) is given by  $\beta = 1 - (1 - \phi)^{3(1 - \phi)}$ .  $\phi$ , porosity;  $\beta_p$  and  $\beta_s$  are the Biot coefficients for the P-wave velocity and S-wave velocity, respectively.

the difference derived from MacBeth is much smaller than that from the proposed equation. For consolidated sedimentary rocks with porosity less than a critical porosity, which is about 0.4 for sandstones (Nur and others, 1998), the difference between  $\beta_p$  and  $\beta_s$  approximately increases as porosity increases.

Note that the application of equations 3 and 4 for bulk and shear moduli implies that the same Biot coefficients are used for bulk and shear moduli; that is,  $\beta_p = \beta_s = \beta$ . Krief and others (1990) proposed a Biot coefficient,  $\beta$ , applicable to equations 3 and 4, and this Biot coefficient is also shown in figure 3B. For porosities less than about 0.2, the Biot coefficient,  $\beta$ , derived from Krief and others (1990) is similar to  $\beta_p$  derived from equation 9; but as porosity increases,  $\beta$  becomes much larger than  $\beta_p$ . However, the Biot coefficient based on the moduli analyzed by MacBeth (2004) agrees better with that from Krief and others (1990). Figure 3B indicates that at porosities less than about 0.2, velocities predicted using the Biot coefficient from Krief and others (1990) are larger than those using the Biot coefficients derived from equations 9 and 10 with  $\alpha = 4$ , and at porosities greater than about 0.25, the opposite is true. Figure 3 implies that, in order to fit the observed data or to accurately predict velocities for a wide range of porosities using the proposed equations (equations 5 and 7), the consolidation parameter should be a function of

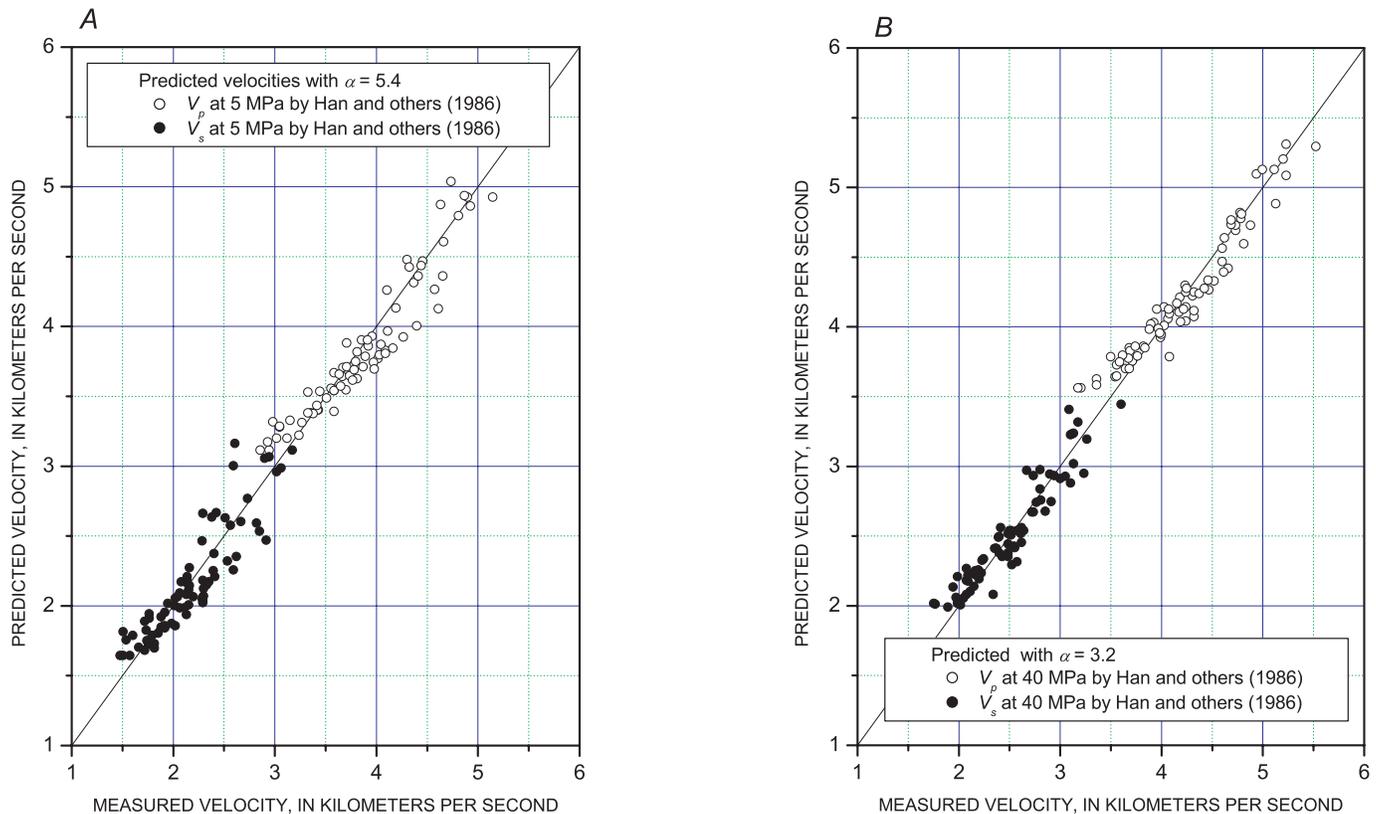
porosity as well as the degree of consolidation and differential pressure.

## Modeling

### Velocities with Respect to Porosity

As indicated previously, for a given rock, the consolidation parameter,  $\alpha$ , controls velocities when using equations 5 and 7. Figure 4 shows predicted velocities of water-saturated sandstones from porosity and dry moduli by equations 5 and 7 and comparisons with measured velocities by Han and others (1986). Figures 4A and 4B show the results of measurements at 5 MPa and 40 MPa, respectively. The fractional errors using  $\alpha = 5.4$  for 5-MPa data are  $0.00 \pm 0.05$ , and  $0.01 \pm 0.08$  for P-wave and S-wave velocities, respectively. If  $\alpha = 4$  is used for the prediction, the fractional errors would be  $0.05 \pm 0.05$  and  $0.09 \pm 0.09$  for P- and S-wave velocities, respectively. This implies that the predicted velocities are highly sensitive to  $\alpha$ .

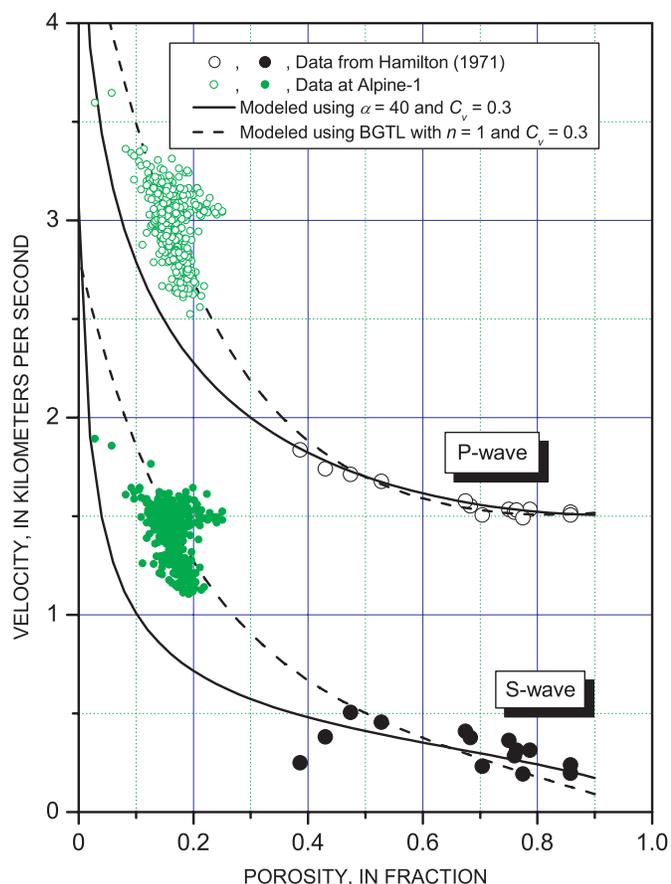
The fractional errors for predicted velocities using  $\alpha = 3.2$  for 40-MPa data are  $0.00 \pm 0.04$  and  $0.01 \pm 0.05$  for P- and S-wave velocities, respectively. As expected, the optimal value of  $\alpha$  for 40-MPa data is less than that for 5-MPa data,



**Figure 4.** Comparison between predicted velocities of water-saturated sandstones, which are computed by applying the Gassmann theory to the dry moduli derived from equations 5 and 7, and measured velocities by Han and others (1986). A, Velocities measured at 5 megapascals (MPa). B, Velocities measured at 40 MPa.  $V_p$ , P-wave velocity;  $V_s$ , S-wave velocity.

and the predicted velocities for 40-MPa data are somewhat more accurate than those for 5-MPa data.

Figure 5 shows modeled velocities for unconsolidated sediments. The large dots and circles represent velocities for deep marine sediments measured by Hamilton (1971), and small circles and dots are well-log velocities acquired at the Alpine-1 well, North Slope of Alaska. Equations 5 and 7 with  $\alpha = 40$  and a volume clay content ( $C_v$ ) of 0.3 predict accurate P- and S-wave velocities for the Hamilton data but underestimate velocities for the well-log data. For comparison, modeled velocities using the modified Biot-Gassmann theory (BGTL) by Lee (2002) with  $n = 1$  and  $C_v = 0.3$  are shown as dashed lines. The predicted velocities for Hamilton data using the BGTL are inferior to those predicted from the proposed equation; but for well-log data at the Alpine-1 well, the BGTL performs better. In order to predict more accurate well-log velocities using equations 5 and 7, a different  $\alpha$ , smaller than 40, could have been used. Results shown in figures 4 and 5 indicate that velocities can be predicted accurately by using



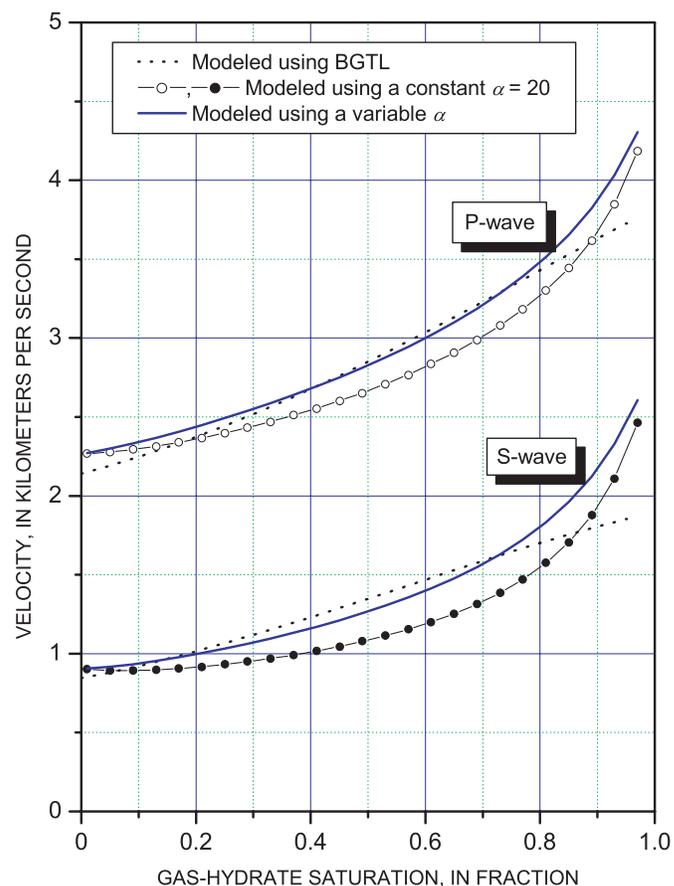
**Figure 5.** Comparison between predicted and measured velocities. Solid lines show velocities predicted by applying Gassmann theory to the moduli of dry frame by equations 5 and 7 with  $\alpha = 40$  and clay volume content of 0.3. Dashed lines represent velocities predicted by the modified Biot-Gassmann theory by Lee (2003; BGTL) with  $n=1$  and clay volume content ( $C_v$ ) of 0.3. Velocities of the Alpine-1 well are from depths of 4,000 to 4,500 feet.

equations 5 and 7, if an accurate consolidation parameter is known. However, the parameter  $\alpha$  is difficult to estimate and is highly variable, as discussed herein.

### Velocities of Gas-Hydrate-Bearing Sediments

Velocities of gas-hydrate-bearing sediments are strongly affected by how gas hydrates interact with porous media, and Helgerud (2001) shows four different cases of gas-hydrate deposits in the pore space. Among four different gas-hydrate-accumulation models, the pore-filling model accurately predicts in-place velocities (Helgerud, 2001; Lee, 2002; Kleinberg and others, 2003).

In the context of the proposed equations (equations 5 and 7), the consolidation parameter  $\alpha$  can be estimated by fitting the measured velocities of non-gas-hydrate-bearing sediment. For a given  $\alpha$ , the velocities of gas-hydrate-bearing sediments can be calculated using the pore-filling model, which only affects the dry-frame bulk and shear moduli of matrix material, which consists of sand, clay, and gas hydrate, as shown in Helgerud (2001) and Helgerud and others (1999).



**Figure 6.** Calculated velocities for gas-hydrate-bearing sediment using various methods. BGTL, modified Biot-Gassmann theory by Lee (2002).

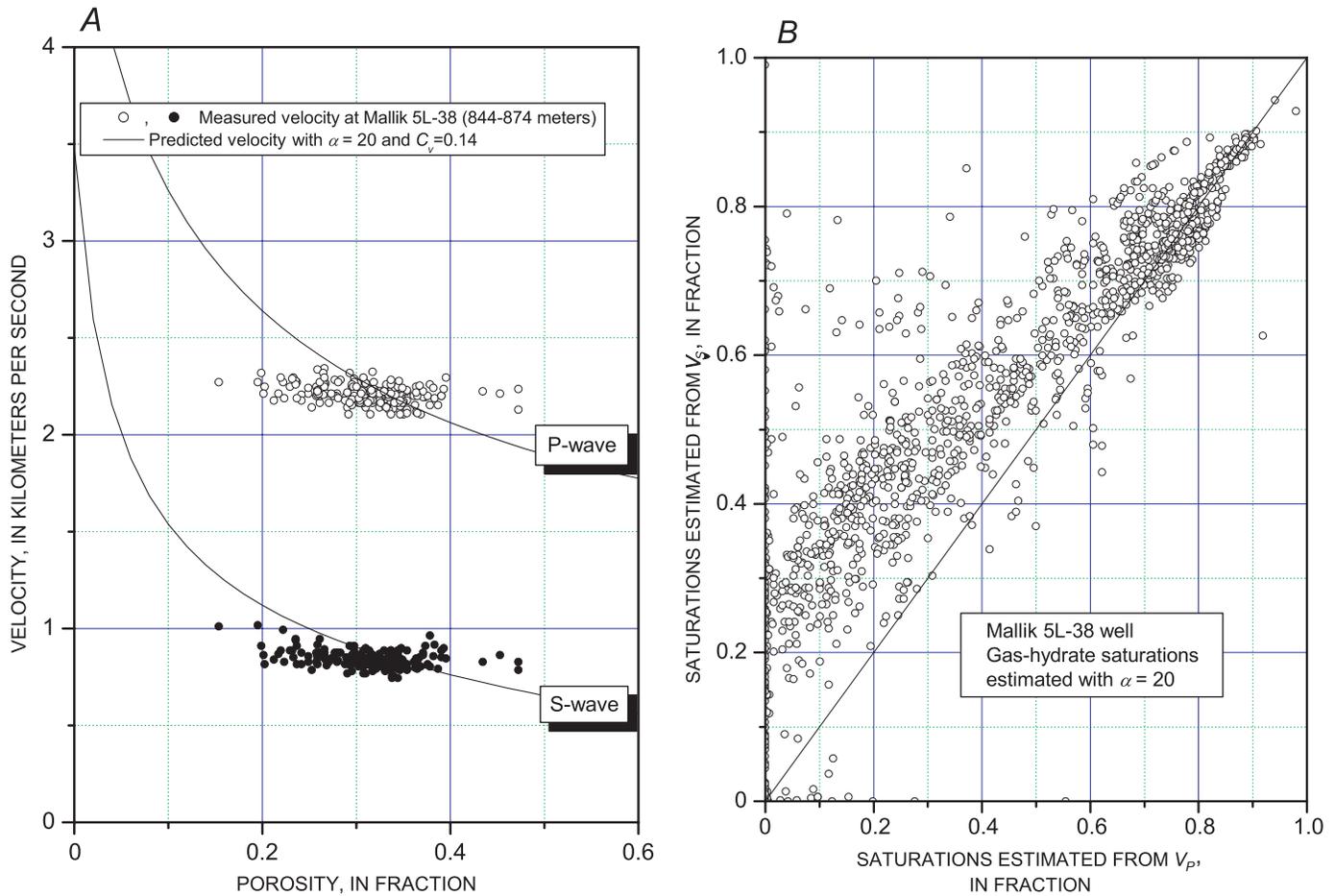
Figure 6 shows calculated elastic velocities with respect to gas-hydrate concentrations for a sediment having  $\phi = 0.32$  and  $C_v = 0.1$ . Velocities calculated from the BGTL (Lee, 2003), with  $n=1$  and  $G = 0.9552 + 0.0448e^{-C_h/0.06714} - 0.18C_h^2$  ( $C_h$  is the gas-hydrate concentration), are shown as dotted lines and serve as reference velocities for comparison. Velocities computed using  $\alpha = 20$  are shown as line-circle-line and are less than reference velocities where gas-hydrate saturations are less than about 0.9. As gas hydrate fills the pore space, the water-filled porosity decreases. Therefore, the consolidation parameter  $\alpha$  should decrease (higher velocities) to fit the reference velocities. The solid lines in figure 6 are computed velocities using the following equation for  $\alpha$  with  $\alpha_0 = 20$ :

$$\alpha = \alpha_0(0.59 + 0.41e^{(-C_h/0.376)}) \quad (18)$$

where  $\alpha_0$  is the consolidation parameter at  $C_h = 0$ . The computed velocities using equation 18 are close to the reference

velocities for gas-hydrate concentrations as much as about 0.8 and become higher than the reference velocities as concentrations increase further. Because the gas-hydrate concentrations are limited by the free water in the pore, the maximum hydrate concentration in sediments is usually less than about 0.9. Therefore, equation 18, with equations 5 and 7, provides accurate velocities for gas-hydrate-bearing sediments.

Figure 7A shows well-log velocities of non-gas-hydrate-bearing sediments recorded at the Mallik 5L-38 well, Mackenzie Delta, in Canada. Modeled velocities with  $\alpha = 20$  agree well with the velocities of non-gas-hydrate-bearing sediments. Figure 7B compares gas-hydrate concentrations estimated from the P-wave velocities and from the S-wave velocities using  $\alpha = 20$ . Except for gas-hydrate concentrations higher than about 0.8, the gas-hydrate concentrations estimated from the P-wave velocities are much less than those from the S-wave velocities, as the modeled velocities shown in figure 6 indicate. Figure 7C shows gas-hydrate concentrations estimated



**Figure 7 (above and facing page).** Velocities and gas hydrate concentrations estimated at the Mallik 5L-38 well, Mackenzie Delta, in Canada. A, Measured velocities and predicted velocities from the proposed equations (equations 5 and 7) with  $\alpha = 20$  for non-gas-hydrate-bearing sediments. B, Gas-hydrate concentrations estimated using the proposed equation. C, Gas-hydrate concentrations estimated using the proposed equation with a variable consolidation parameter, which decreases with increasing gas-hydrate concentration.

using equation 18. Gas-hydrate concentrations estimated from P-wave velocities are similar to those estimated from S-wave velocities.

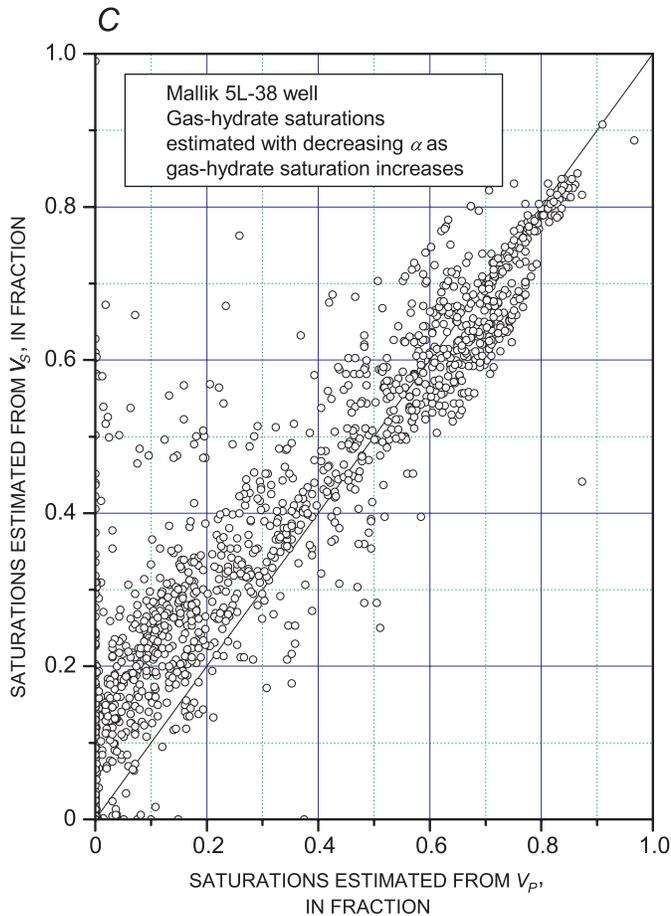
## Results and Applications

### Comparison with Other Moduli

The contact theory (Digby, 1981; Murphy and others, 1993; Winkler, 1983) predicts that the ratio of bulk to shear moduli for dry rock is independent of porosity but dependent on differential pressure. Contact theory predicts that the velocity ratio ( $V_p/V_s$ ) decreases as differential pressure increases. Application of equations 5 and 7 predicts the same relation. Equations 5 and 7 also predict that the  $V_p/V_s$  ratio increases as porosity increases, but this differs from the prediction of the contact theory.

Murphy and others (1993) suggested the following approximate linear relation between the frame and matrix moduli for sandstones at high differential pressure:

$$k_d = k_{ma}(1 - 2.5\phi) \text{ and } \mu_d = \mu_{ma}(1 - 2.5\phi). \quad (19)$$



According to Murphy and others (1993), the velocity ratio of dry rock at high differential pressure is independent of porosity, which is the same as the prediction of Krief and others (1990). The magnitudes of Biot coefficient estimated from the least-squares fit (LSF) to data analyzed by MacBeth (2004), which are shown in figure 3, are similar to those from Murphy and others (1993). Even though both the proposed method and the MacBeth (2004) data indicate  $\beta_p < \beta_s$ , the difference between  $\beta_p$  and  $\beta_s$  for the MacBeth data is negligible.

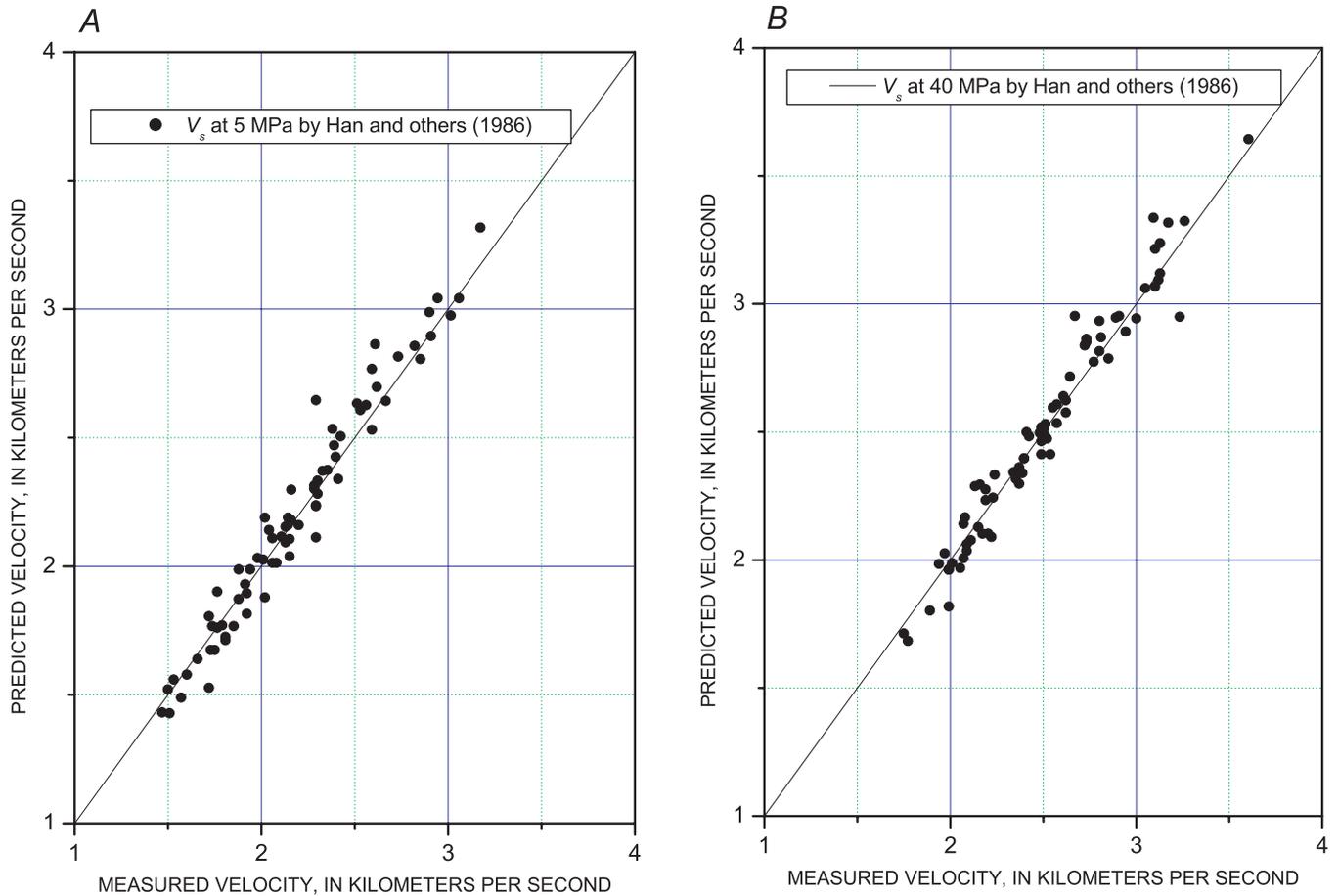
The Kuster and Toksöz (1974) theory predicts that the velocity ratio increases as porosity increases. Therefore, the prediction based on equations 5 and 7 is similar to the Kuster and Toksöz theory for the relation between porosity and velocity ratio and is similar to the contact theory for the relation between velocity ratio and differential pressure. The MacBeth (2004) formula predicts increasing, constant, decreasing, or alternate velocity ratios with respect to differential pressure. However, for most rocks, the velocity ratio increases with pressure. The discrepancy between the observation by MacBeth (2004) and the prediction of the proposed equation comes from differences in pore geometry. Sandstones containing microcracks behave differently from rocks containing intergranular porosities. As differential pressure increases, small microcracks close rapidly. Consequently, the P-wave velocity increases rapidly as differential pressure increases, resulting in increasing  $V_p/V_s$  with increasing differential pressure. Therefore, equations 5 and 7 work well for sedimentary rocks having intergranular porosities and less accurately for those having microcracks.

### Predicting S-Wave Velocity from P-Wave Velocity and Porosity

Use of equations 5 and 7 provides a means for accurately predicting S-wave velocities of water-saturated sandstones from P-wave velocity and porosity because a single parameter  $\alpha$  relates both bulk and shear moduli of dry frame. Let us define the predicted P-wave velocity using the BGT with the dry moduli derived from equations 5 and 7 as  $V_p^*$  and as  $V_p^m$  for measured P-wave velocity. The consolidation parameter can be calculated by solving the following equation:

$$V_p^*(\alpha) - V_p^m = 0. \quad (20)$$

Figure 8 shows the predicted S-wave velocities for sandstones measured at 5 MPa and 40 MPa by Han and others (1986) by solving equation 20 using the Newton-Raphson method (Press and others, 1986). The fractional errors for predicted S-wave velocities are  $0.01 \pm 0.04$  and  $0.00 \pm 0.04$  for 5-MPa and 40-MPa data, respectively. Figure 9 shows the predicted S-wave velocities for unconsolidated sediments at the Alpine-1 well, North Slope of Alaska. The fractional error for S-wave velocities is  $-0.01 \pm 0.04$ .



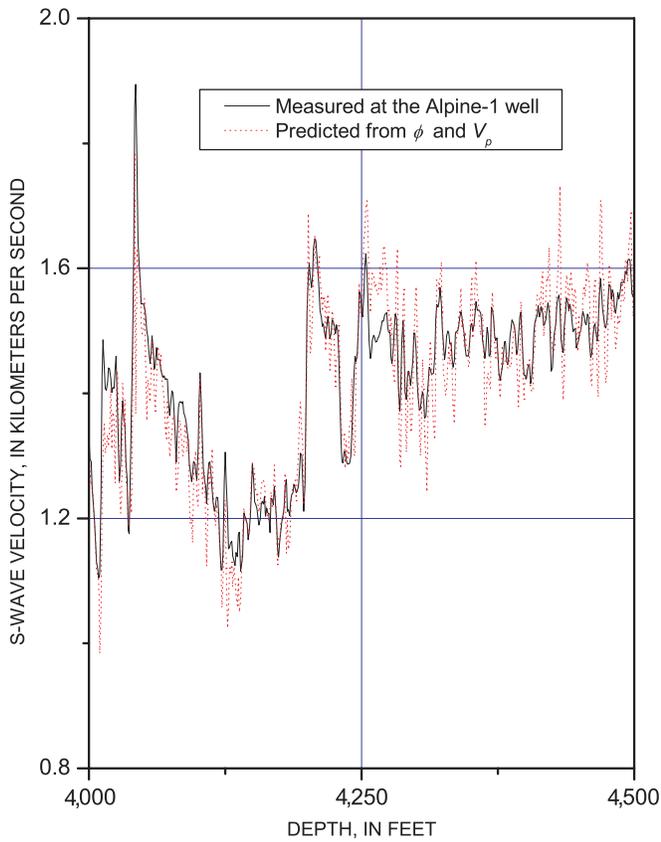
**Figure 8.** Comparison between the predicted S-wave velocity, which is calculated from the P-wave velocity and porosity using the dry-rock moduli from equations 5 and 7, and measured S-wave velocity by Han and others (1986). A, Velocities measured at 5 megapascals (MPa). B, Velocities measured at 40 MPa.  $V_s$ , S-wave velocity.

The accuracy of predicted S-wave velocities for dry rock is different from that of water-saturated rock. Figure 10 shows the predicted S-wave velocities of dry rock measured by Domenico (1977) and Gregory (1976), using equation 7 for the shear moduli. The fractional error of the predicted S-wave velocity is  $-0.12 \pm 0.01$  for the Domenico data and  $-0.08 \pm 0.04$  for the Gregory data. Both results indicate a large underestimation of S-wave velocities. The line and star in figure 10 shows the predicted S-wave velocities using  $\gamma = 1$  in equation 7. The fractional errors using  $\gamma = 1$  are  $0.00 \pm 0.01$  and  $0.00 \pm 0.04$  for the Domenico data and the Gregory data, respectively. Using  $\gamma = 1$  is the same as using identical Biot coefficients for P- and S-wave velocities, which is predicted by Murphy and others (1993) and Krief and others (1990). The result shown in figure 10 indicates that the proposed moduli are accurate when calculating velocities of water-filled sedimentary rocks. However, use of  $\gamma = 1$  for

dry rock appears to be more accurate. The Biot coefficients based on the LSF to moduli at infinite pressure derived from 179 sets of data by MacBeth (2004), as shown in figure 3B, indicate that  $\gamma = 1$  is a close approximation for shear modulus of sandstones at high differential pressure.

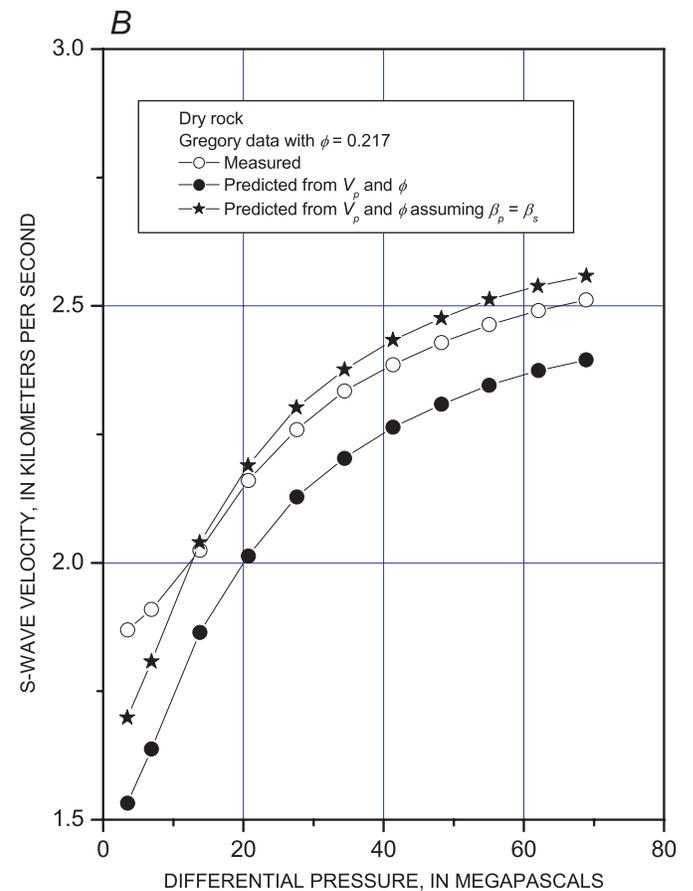
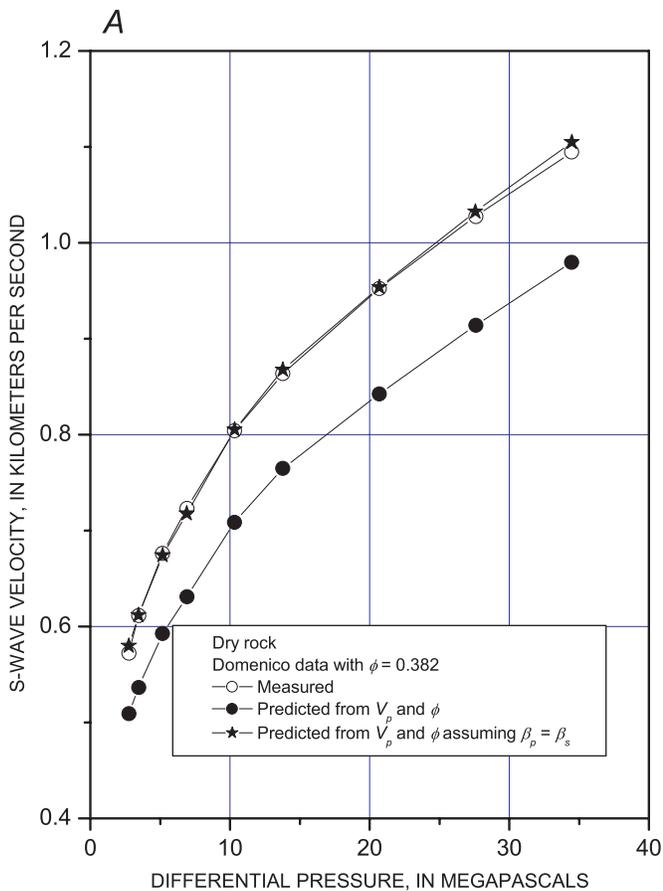
### Consolidation Parameter $\alpha$

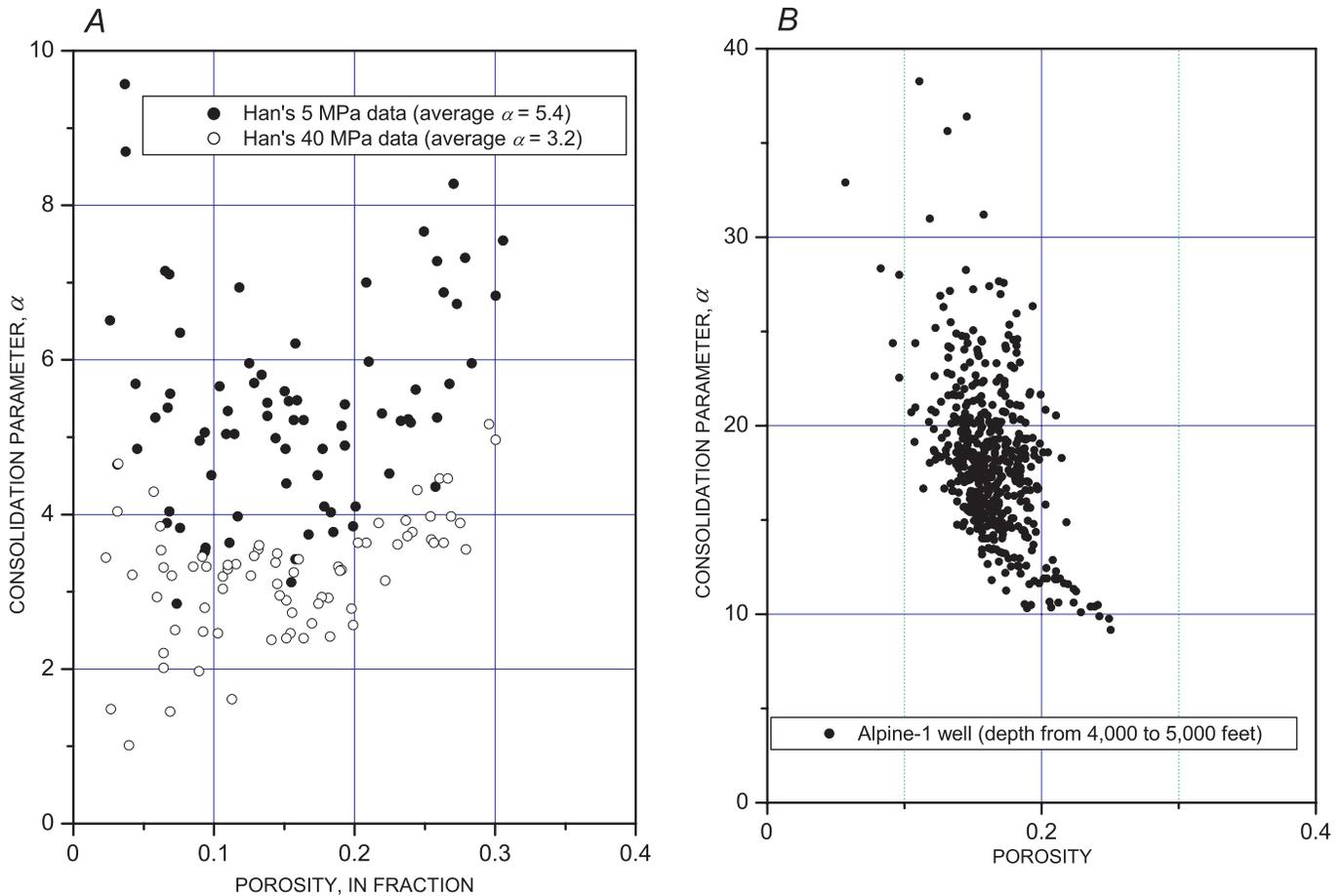
When velocities are calculated using equations 5 and 7,  $\alpha$  is the only parameter to choose. The general behavior of  $\alpha$  is known. For example,  $\alpha$  becomes small as the degree of consolidation and differential pressure increases. But the precise value of  $\alpha$ , on the other hand, is not known and should be estimated from the data or assumed for a given sedimentary rock. In the case that P- and (or) S-wave velocities with porosity are known, the parameter  $\alpha$  can be estimated by solving equation 20.



**Figure 9 (left).** Comparison between the predicted S-wave velocity, which is calculated from the P-wave velocity and porosity using the dry rock moduli from the proposed equation, and the measured well-log velocity at the Alpine-1 well, North Slope of Alaska.  $\phi$ , porosity;  $V_p$ , P-wave velocity.

**Figure 10 (below).** Measured and predicted S-wave velocities for dry rocks. Measured velocities are shown as line-circle-line. Line-dot-line indicates the predicted S-wave velocity ( $V_s$ ) using equation 7, and line-star-line represents for the predicted S-wave velocity using equation 7 with  $\gamma = 1$ . *A*, Unconsolidated dry sediment measured by Domenico (1977). *B*, Semiconsolidated dry rock measured by Gregory (1976).  $\phi$ , porosity;  $V_p$ , P-wave velocity;  $\beta_p$  and  $\beta_s$  are the Biot coefficients for the P-wave velocity and S-wave velocity, respectively.





**Figure 11.** Estimated consolidation parameters from the P-wave velocity and porosity. *A*, Estimated from velocities of sandstones measured by Han and others (1986). *B*, Estimated from velocities of well-log data acquired at the Alpine-1 well, North Slope of Alaska. MPa, megapascals.

Figure 11 shows the estimated  $\alpha$ , which is derived from the porosity and P-wave velocity when predicting S-wave velocities shown in figures 8 and 9. Figure 11A, based on data from Han and others (1986), indicates that, for a given set of sandstones, the consolidation parameter  $\alpha$  varies between 2 to 10 for 5-MPa data and between 1 and 5 for 40-MPa data. The consolidation parameters  $\alpha$  of 5.4 and 3.2 used for figure 4 are the average values of  $\alpha$  shown in figure 11A. Figure 11B, using data from the Alpine-1 well, indicates that  $\alpha$  varies between around 10 and 30, even though the depth range is only 4,000 to 4,500 ft. Figure 11 implies that, although  $\alpha$  is related to differential pressure and degree of consolidation, determining an accurate  $\alpha$  is not simple; thus, it can be considered as a convenient free parameter to fit observed velocities. Pride (2005) suggested that if  $\alpha$  is greater than 20 or 30, moduli predicted by Walton (1987) are preferred. However, the prediction of P- and (or) S-wave velocities shown in figures 5 through 9 indicates that the moduli derived from the

proposed equations work well for consolidated rocks as well as for unconsolidated sediments.

## Conclusions

By applying the BGT to moduli of dry frame proposed here, elastic velocities of water-saturated sandstones rocks can be accurately predicted. The following conclusions are drawn from the present study:

1. The Biot coefficient estimated from the P-wave velocity is smaller than that estimated from the S-wave velocity; this behavior is similar to the prediction of MacBeth (2004) but different from that of Murphy and others (1993) or Krief and others (1990).
2. Contrary to the prediction of MacBeth (2004), the velocity ratio,  $V_p/V_s$ , decreases as differential pressure increases, similar to the prediction of contact theory. This discrepancy is caused by microcracks present in the measured

sandstones, and moduli predicted by the proposed equation are optimal for sandstones with granular porosities.

3. The  $V_p/V_s$  ratio increases as porosity increases, which is similar to the prediction of Kuster and Toksöz (1974) but different from Krief and others (1990), who predicted that the velocity ratio is independent of porosity.

4. Accurate S-wave velocities of water-saturated sandstones can be predicted because the consolidation parameter is related to both P- and S-wave velocities. Parameter  $\alpha$  can be estimated if either P- or S-wave velocity, as well as porosity, is known.

5. Although velocities of water-saturated sandstones can be predicted accurately, based on the proposed equations, predicting velocities of gas-hydrate-bearing sediments by using the proposed equation is not accurate. However, by making the consolidation parameter  $\alpha$  decrease with increasing gas-hydrate concentration, accurate gas-hydrate concentrations (or velocities) can be estimated.

6. For a dry rock, the proposed equation is not accurate. However, by using  $\gamma = 1$ , accurate velocities, particularly at high differential pressure, can be predicted.

7. Parameter  $\alpha$  is related to the degree of consolidation and differential pressure. The exact behavior, however, is difficult to predict. In practical applications, parameter  $\alpha$  can be viewed as a free parameter to fit the observation.

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